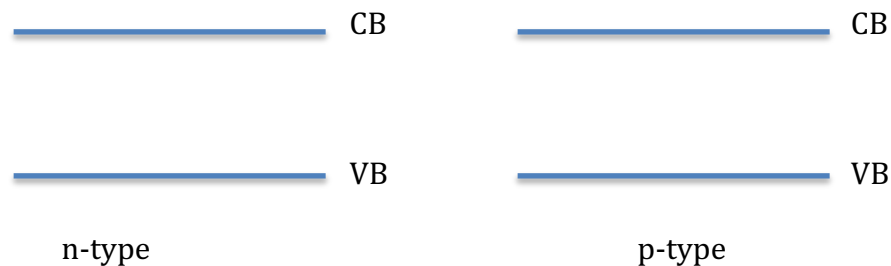


Homework 4 – Doping and the PN junction
due 5 November 2018

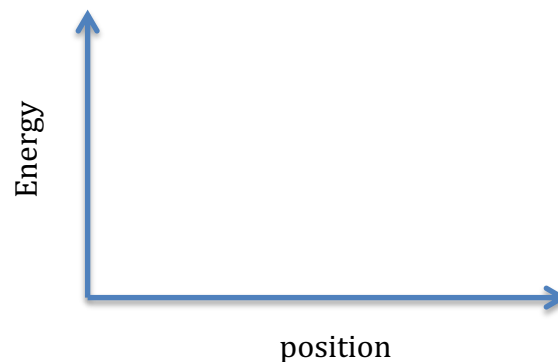
For this homework, you will use the concepts of doping, and the PN junction in the dark and under illumination, and solar cell IV curves.

Question 1.

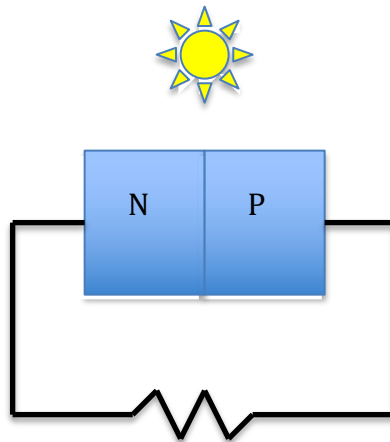
- a) Say we are constructing a PN junction from doped silicon. The n-type material is doped with phosphorous at a concentration of $(6 \times 10^{18}) \text{ cm}^{-3}$ and the p-type material is doped with boron at a concentration of $(4 \times 10^{16}) \text{ cm}^{-3}$. Calculate the Fermi level (chemical potential) for each material (before the junction is formed). For each material, draw Fermi level and label distances in eV from the conduction and valence bands on the two diagrams below. For this calculation, assume that the effective density of states for the conduction and valence bands are, respectively, $N_c = 2.8 \times 10^{19} \text{ cm}^{-3}$ and $N_v = 1.8 \times 10^{19} \text{ cm}^{-3}$.



- b) The two materials are now brought into contact with each other to form the PN junction. Draw the resulting band diagram, and label the Fermi energy, valence band, conduction band, and space charge region.



- c) What is the built-in voltage for the junction (that is, the difference in the chemical potentials between the two sides)?
- d) What is the width of the space charge region at zero applied voltage? What is the width at -0.5 and +0.5 applied volts?
- e) Plot the current density versus applied voltage from -1.0 to 1.0 volts in the dark. Label the saturation current density. What expressions did you use to make the plot? You can assume a small saturation current density around 0.01 mA/cm^2 .
- f) Now, we turn on the light. Under illuminated conditions, assume that the short circuit current is 0.0425 A/cm^2 (this is an ideal value, calculated from the AM1.5G spectrum, assuming that all photons of energy $E_{\text{ph}} > E_{\text{g}}$ are absorbed and collected). Re-plot the IV curve from the previous part.
- g) Show the direction of the current flow through the illuminated device on the diagram below (recall: current is defined as the flow of positive charge).



Homework #2 Solutions

(1) Fermi levels:

n-type material:

$$E_F = E_c + kT \ln\left(\frac{N_D}{N_c}\right)$$

$$= E_c + (26 \text{ meV}) \ln\left[\frac{6 \times 10^{18}}{2.8 \times 10^{19}}\right]$$

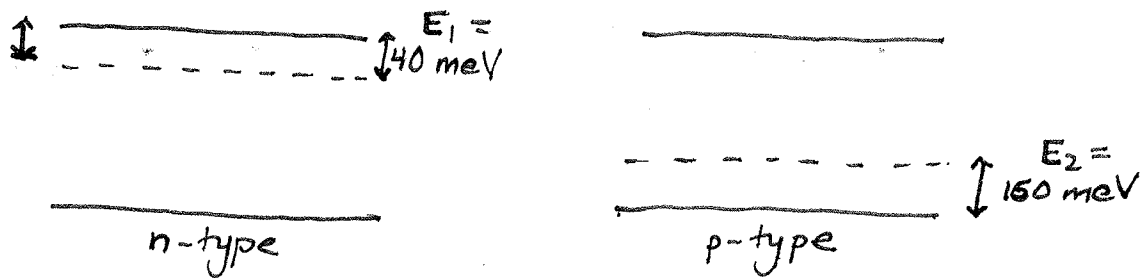
$$= E_c - 40.05 \text{ meV}$$

p-type material:

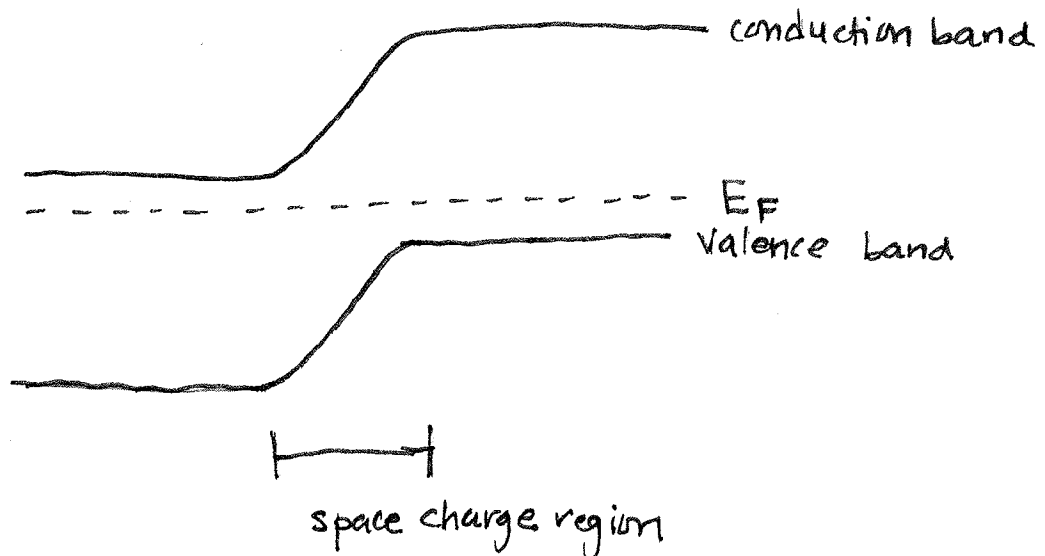
$$E_F = E_v - kT \ln\left(\frac{N_A}{N_v}\right)$$

$$= E_v - (26 \text{ meV}) \ln\left(\frac{4 \times 10^{16}}{1.8 \times 10^{19}}\right)$$

$$= E_v - 158.84 \text{ meV}$$



(2) When brought into contact



(3) built-in voltage ψ_0

$$q\psi_0 = E_g - E_1 - E_2$$
$$= (1.1 - 0.040 - 0.150) \text{ eV}$$

$$\psi_0 \Rightarrow 0.91 \text{ V}$$

(4) space charge width is given by

$$W = \left[\frac{2\epsilon}{q} (\psi_0 - V) \cdot \left(\frac{1}{N_A} + \frac{1}{N_D} \right) \right]^{1/2}$$

using $\epsilon = \epsilon_r \epsilon_0$ with $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$
 $\epsilon_r = 11.7$

$$W = \left[\frac{2 \times (11.7 \times 8.85 \times 10^{-12} \text{ F/m})}{1.6 \times 10^{-19} \text{ C}} * (0.91 - V_A) * \left(\frac{1}{4 \times 10^{22} \text{ m}^{-3}} + \frac{1}{6 \times 10^{24} \text{ m}^{-3}} \right) \right]^{1/2}$$

$$= (1.8048 \times 10^{-7}) * \sqrt{0.91 - V_A} \text{ m}$$

for $V_A = 0$	$W = 172 \text{ nm}$
$+ 0.5$	$W = 115 \text{ nm}$
$- 0.5$	$W = 214 \text{ nm}$

(5) The IV curve in the dark is given by the ideal diode equation $I = I_0 (e^{qV/nkT} - 1)$

use a saturation current around 0.0001
density

see attached plot

(6) With the light on, the IV characteristic becomes

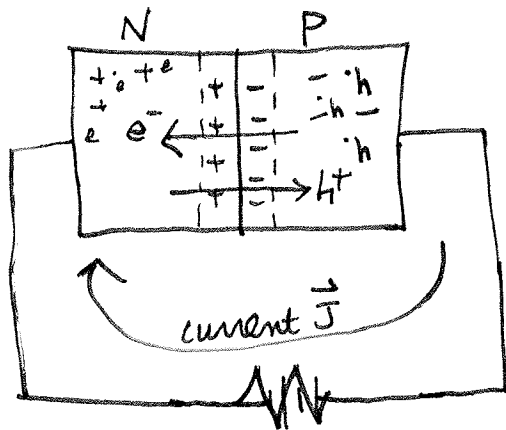
$$I = I_{\text{dark}} - I_{\text{ill}}$$

$$= I_0 (e^{qV/kT} - 1) - I_{\text{ill}}$$

$$I_0 = 0.0001 \text{ mA}$$

$$I_{\text{ill}} = 0.0425 \text{ A} = 42.5 \text{ mA}$$

(7) Upon illumination, $e^- - h^+$ pairs are created and the minority carriers near the depletion region are swept across the junction (drift current). We have



Question 2. By using the ideal diode equation, draw how the IV curve will change at solar cell operating temperature $T=340$ K vs. room temperature (300 K). Which terms in the ideal diode equation are affected by temperature, and how? Why does the performance of the solar cell decrease at higher temperatures?

Noting that we have

$$J = J_o \left(\exp \left[\frac{qV}{kT} \right] - 1 \right) + J_{ill} \quad ; \quad J_o = \left(\frac{qD_e n_i^2}{L_e N_A} + \frac{qD_h n_i^2}{L_h N_D} \right)$$

Several of the terms in the equations above have a temperature-dependence. Of course, the diffusion component of the dark current has an explicit exponential dependence on temperature. However, J_o also has a dependence on temperature: in the expression for J_o , the diffusion coefficient D , the intrinsic carrier concentration n_i , and the minority carrier diffusion length L all depend on temperature. The biggest single dependence however appears for the intrinsic carrier concentration, and considering how n_i varies with T is sufficient to capture the main effect that we observe in the real world. We have:

$$n_i = C \exp \left(-\frac{E_G}{2kT} \right) \quad ; \quad J_o = C' \exp \left(-\frac{E_G}{kT} \right)$$

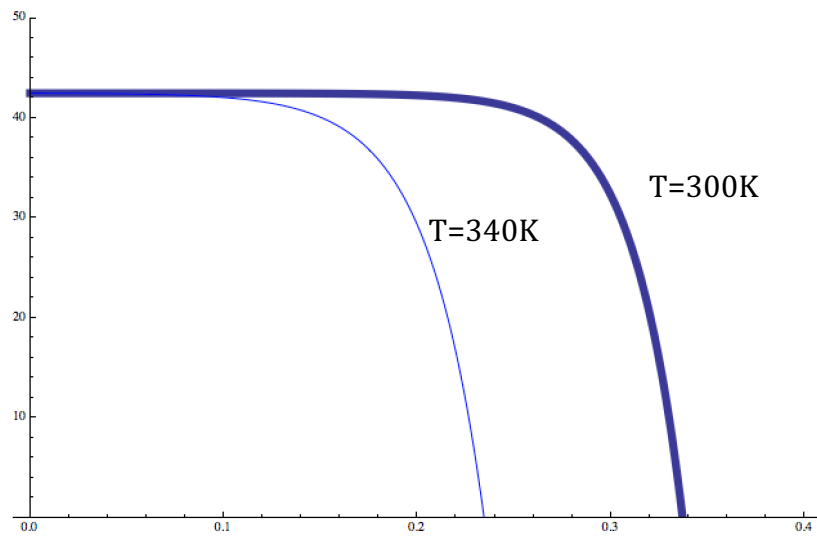
Thus, we need to see how the saturation current J_o changes from its value at $T=300$ K to its new value at $T=340$ K. Since we know that $J_o = 0.01$ mA/cm² at $T=300$ K, we can solve for the constant C' .

$$\begin{aligned} J_o (@300K) &= C' \exp \left(-\frac{E_G}{kT} \right) \\ 0.01 &= C' \exp \left(-\frac{1.1 \text{ eV}}{8.6173(10^{-5}) \text{ eV/K} \cdot 300 \text{ K}} \right) \\ C' &= 3.0148(10^{16}) \text{ mA/cm}^2 \end{aligned}$$

And now we can find the new value of J_o at $T=340$ K:

$$\begin{aligned} J_o (@340K) &= C' \exp \left(-\frac{E_G}{kT} \right) \\ &= C' \exp \left(-\frac{1.1 \text{ eV}}{8.6173(10^{-5}) \text{ eV/K} \cdot 340 \text{ K}} \right) \\ &= 1.49 \text{ mA/cm}^2 \end{aligned}$$

Note how substantially a mere 40C increase in temperature raises the saturation current! Now we can compare the two JV curves directly. See how much the open circuit voltage drops due to the higher temperature.



Question 3. Now we'll calculate many of the performance parameters of a solar cell given both the dark and illuminated I-V curves. Assume the illumination is done under AM1.5G spectrum. The I-V curves are posted online. The measured cell has an area of 3 cm^2 , and that the data was taken at room temperature.

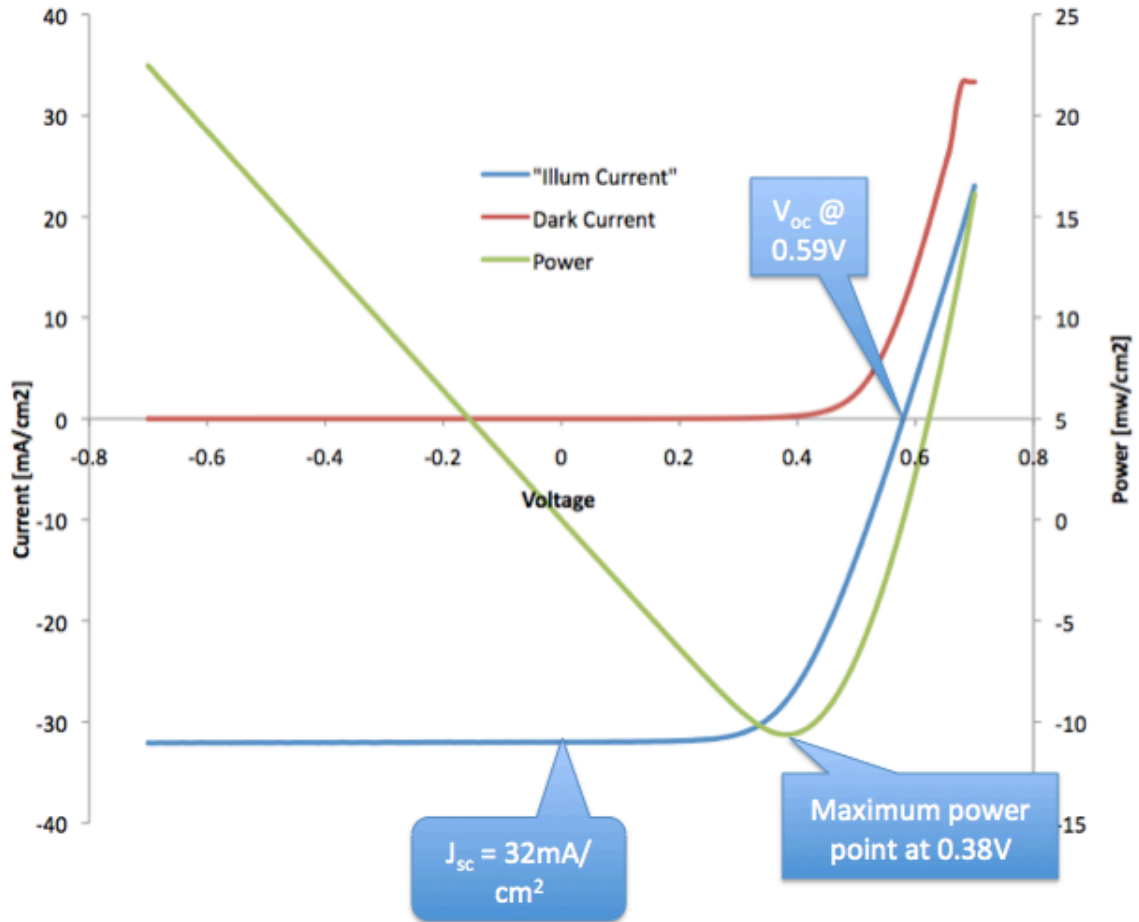
- a) Plot the following on a single chart with two different y-axes (one for current density [mA/cm^2], the other for output power [mW/cm^2]):
 - i. Illuminated J-V curve
 - ii. Dark J-V curve
 - iii. Power output of illuminated device as a function of voltage
- b) On the plot created in (a), please label the following:
 - i. Open-circuit voltage (V_{OC})
 - ii. Short-circuit current density (J_{sc})
 - iii. Maximum power point
- c) For the cell, please calculate parameters listed below. For each calculation show the formula used if you use one.
 - i. Open-circuit voltage
 - ii. Short-circuit current
 - iii. Voltage at maximum power point
 - iv. Current at maximum power point
 - v. Fill Factor
 - vi. Efficiency

Answer Key for HW #3
Joseph Sullivan

Problem 1 (20 pts)

a and b)

In the graph below, you can either report positive current as current out or negative current as current out (i.e. if you flip the y-axis, it is still correct!)



c)

$$V_{oc} = 0.59V$$

$$J_{sc} = 32mA/cm^2$$

$$V_{mpp} = 0.38V$$

$$J_{mpp} = 28mA/cm^2$$

$$FF = \frac{V_{mpp} * J_{mpp}}{V_{oc} * J_{sc}} = 56\%$$

$$Efficiency = \frac{V_{mpp} * J_{mpp}}{\Phi} = 10.7\%$$

Note that the insolation, Φ , is 100mW/cm² for AM1.5G.