TAM 451: Intermediate Solid Mechanics  
Homework 2 - Due Monday September 25th

**Question 1:** A slender bar of material of length $2\ell$ (like a propeller) is made to spin at a constant angular velocity $\omega$ about an axis perpendicular to the bar at its midpoint. The mass density of the material is $\rho$. In a coordinate system fixed with respect to the rotating bar, the material of the bar appears to be in equilibrium under the action of a body force acting along the length of the bar due to inertia. Let the $x_1$ axis be aligned with the bar and assume all the stress components are zero except $\sigma_{11}$. The inertial body force is

$$X_1 = \rho \omega^2 x_1$$

a) Check the physical units to verify that $X_1$ is indeed a force per unit of volume.
b) What is the traction acting on the ends of the bar $x_1 = \pm \ell$, and what boundary conditions on $\sigma_{11}(x_1)$ does this imply?
c) Determine the stress distribution in the bar from the equilibrium field equation.
d) Suppose that $\ell = 0.3$ m, $\rho = 7800$ kg/m$^3$ (steel), and $\omega = 10^3$ rad/s ($\sim 3000$ rpm). Plot the stress vs. position in the bar. What is the maximum stress in the bar?
e) Suppose the material is replaced with one of mass density 4500 kg/m$^3$ (titanium). How fast can the bar spin to attain the same maximum stress as in part (d)?

**Question 2:** The stress components in the wall of a cylindrical tank (closed ends) of thickness $h$ and radius $r$, due to internal pressure $p$, are.

$$\sigma_{11} = \frac{pr}{2h} \quad \sigma_{22} = \frac{pr}{h}$$

in the axial and circumferential directions, respectively. All other stress components are zero (or at least negligible by comparison). What is the maximum shear stress in the material?

**Question 3:** The state of stress in a metal being tested for its shear response under high pressure is

$$\sigma_{11} = -100 \text{ MPa}$$
$$\sigma_{22} = -200 \text{ MPa}$$
$$\sigma_{33} = -100 \text{ MPa}$$
$$\sigma_{12} = \sigma_{21} = 80 \text{ MPa}$$

and all other stress components are zero.

a) Determine the principal stresses for this state of stress.
b) Determine the corresponding principal directions. Show the orientation of the principal axes in a sketch.
**Question 4:** In its initial configuration, a block of material occupies the region $0 \leq x_1, x_2, x_3 \leq 1$. The block undergoes a displacement specified by the field $u(x_1, x_2, x_3)$, where

$$
\begin{align*}
    u_1 &= -x_1 - x_2 + 1 \\
    u_2 &= x_1 - x_2 \\
    u_3 &= 0
\end{align*}
$$

a) Determine the current geometry and show it in a sketch.
b) Determine all components of the Lagrange strain tensor $E_{ij}$.
c) Determine all components of the small strain tensor $\epsilon_{ij}$.
d) Why are the two strain tensors different? Is this a case of small deformation?