Linear Elastic Fracture Mechanics - study of crack formation & propagation in materials

* we want a framework to model material failure directly
* brittle or ductile fracture is the most common failure mechanism

Note: There are three ways to apply a force to enable a crack to propagate:

**Mode I:** Opening
- tensile load is normal to crack plane

**Mode II:** In-plane shear sliding
- shear is parallel to crack plane, 1 to crack front

**Mode III:** Out of Plane Shear / Tearing
- shear is parallel to crack plane, parallel to crack front

**Key Question:** Under what conditions will the applied loads cause the crack to grow?

A common model based on thermodynamic arguments is Griffith theory
Griffith approach compares total energy before and after growth of crack. If energy is reduced by crack growth, then it will grow (and v.v.)

* regard the two triangular regions near crack flank as being completely unloaded, while remaining material feels the full stress \( \sigma \)

* length \( \beta \) is determined by boundary conditions -- for plane stress solution is \( \beta = \pi \)

\[ \rightarrow \text{assume linear elastic model (E, v)} \text{ and free surface energy of } \gamma \text{' (energy/length)} \]

\[ \rightarrow \text{in loaded region strain energy density is } \sigma^2 / 2E \]

If the crack of length \( a \) forms, it

1. reduces strain energy by
   \[ \frac{\sigma^2}{2E} \frac{1}{2} (2\beta a) a = \frac{\sigma^2}{2E} \pi a^2 \]

2. increases the surface energy by \( 2a \gamma \)

The total change in energy is

\[ \Delta V_{tot} = 2a \gamma - \frac{\sigma^2}{2E} \pi a^2 \]
We have:

\[
\frac{d}{da} \left( \Delta V_{\text{tot}} \right) = 0
\]

\[a = a_c\]

\[0 = 2\gamma - \frac{\sigma^2 \pi a_c}{E}\]

\[a_c = \frac{2\gamma E}{\sigma^2 \pi}\]

\[\sigma_c = \sqrt{\frac{2\gamma E}{\pi a}}\]

"Griffith Criterion"

or generally crack will grow if

\[\sigma \sqrt{\pi a} > \sqrt{2\gamma E}\]
Origin of Cracks: cracks originate at stress concentrations -- i.e. pores, scratches, notches, sharp corners where the stress is locally amplified within the component.

By convention we define the degree of amplification by the stress intensity factor $K$

$$K = Y \sigma \sqrt{\pi a}$$

$Y$ geometrical factor, dimensionless

e.g.) for a crack of length $2a$ inside an infinite plate, it can be shown that $Y = 1$. Thus

$$K = K_{IC} = \sigma \sqrt{\pi a} = \sqrt{2\pi E \sigma}$$

mode I "critical"

If in the material $\sigma \sqrt{\pi a} > K_{IC}$, we are in the region of unstable crack growth (where energy released by growth > energy required for growth)

$K_{IC}$ is a material property ($K_{IC} = \sqrt{2\pi E}$), sometimes called "fracture toughness"

typical values: $Y = 1 \text{ J/m}^2$, $E = 100(10^9) \text{ Pa}$

$$\Rightarrow K_{IC} = \frac{1}{2}(10^6) \text{ Pa}\sqrt{m}$$
How well does Griffith theory work?

In experiments:

* Works well for brittle materials where surface formation is the only energy required for crack growth (i.e., glass)
* For ductile materials, we measure $K_{IC} \sim (100)(10^6)$ Pa $\sqrt{m}$ because energy is also dissipated near the crack tip via plastic deformation (i.e., plastic deformation reduces the stress intensity at the crack tip substantially)

For such materials, we adopt a correction due to Irwin (1956):

$$K_{IC}^{\text{Irwin}} = \sqrt{2\gamma_E E + E \gamma_p} = \sqrt{GE}$$

\[ G = 2\gamma_E + \gamma_p \] total energy release rate

Generalized Criterion: $\sigma \sqrt{\pi a} = \sqrt{EG}$

* For brittle materials $G \approx 2\gamma \approx 2 \text{ J/m}^2$
* For ductile materials $G \approx \gamma_p \approx 1000 \text{ J/m}^2$

What do the stress fields around a crack tip look like?

Irwin also showed that we can express

$$\sigma_{ij} \approx \frac{K}{\sqrt{2\pi r}} f_{ij}$$

(5)

$K$ = stress intensity factor, depends on shape of sample &
Example  DeHavilland Comet - early 1950's

- fuselage made of clad aluminum  $E = 1 \times 10^6$ psi
- effective energy release rate $G \approx 300$ in-psi
- hoop stress due to relative cabin pressurization 20,000 psi

$\Rightarrow$ critical crack length

$$a_{cr} = \frac{GE}{\sigma^2 \pi} = \frac{(300)(11 \times 10^6)}{\pi (20000)^2} = 2.62 \text{ inches}$$

Why wasn't a crack this large detected during routine inspection? Cracks were propagating from rivet holes near cabin windows. When a crack reached the window, the size of the window opening was effectively added to the crack length.