lowed energy levels introduced by a range of impurities in the silicon as well as the compound semiconductor GaAs. Some impurities introduce multiple energy levels, as shown. Crystal defects act in a similar way to introduce allowed levels into the forbidden gap.

Impurities, particularly those which introduce energy levels near the middle of the band gap, generally degrade the properties of semiconductor devices. Impurity concentrations in the starting material used in the fabrication of these devices are therefore kept as low as technology will allow—generally less than 1 part per billion.

### 2.14 CARRIER TRANSPORT

#### 2.14.1 Drift

Under the influence of an applied electric field, \( \xi \), a randomly moving free electron would have an acceleration \( a = \xi/m \) in a direction opposite to the field, with its velocity in this direction increasing with time. The electron in a crystal structure is in a different situation. It moves with a different mass and will not continue accelerating for very long. It will eventually collide with a lattice atom, or an impurity atom, or a defect in the crystal structure. Such a collision will tend to randomize the electron's motion. In other words, it will tend to reduce the excess velocity that the electron picked up in the applied field. The “average” time between collisions is called the relaxation time, \( t_r \). This will be determined by the random thermal velocity of electrons, which is generally much larger than field imparted velocities. The average velocity increase of electrons between collisions caused by the field is called the drift velocity and is given by

\[
v_d = \frac{1}{2} at = \frac{1}{2} \frac{qt_r}{m_e^*} \xi
\]

(2.31)

for electrons in the conduction band. (The factor of 2 disappears if \( t_r \) is averaged over all electron velocities.) The electron carrier mobility is defined by the ratio

\[
\mu_e = \frac{v_d}{\xi} = \frac{q t_r}{m_e^*}
\]

(2.32)

The corresponding current density flow due to conduction band electrons will be

\[
J_e = q n v_d = q n \mu_e \xi
\]

(2.33)

An analogous equation for holes in the valence band is

\[
J_h = q \mu_h p \xi
\]

(2.34)

The total current flow is just the sum of these two components. Hence, the conductivity, \( \sigma \) of the semiconductor can be identified as

\[
\sigma = \frac{1}{\rho} = \frac{J}{\xi} = q \mu_e n + q \mu_h p
\]

(2.35)

where \( \rho \) is the resistivity.

Although the analysis resulting in Eq. (2.32) is simplistic, it does allow an intuitive understanding of how the carrier mobilities, \( \mu_e \) and \( \mu_h \), change with changes in the density of dopants, temperature, and electric field strength.

For relatively pure semiconductors of good crystallographic quality, the collisions that randomize the carrier velocities will involve the atoms of the host crystal. However, ionized dopants are very effective scatterers because of their associated net charge. Consequently, as the semiconductor becomes more heavily doped, the average time between collisions and hence the mobility will decrease. For good-quality silicon, empirical expressions relating the carrier mobilities to the level of dopants \( N \) (in \( \text{cm}^{-3} \)) are (Ref. 2.5)

\[
\mu_e = 65 + \frac{1265}{1 + (N/8.5 \times 10^{16})^{0.72}} \text{cm}^2/\text{V-s}
\]

\[
\mu_h = 47.7 + \frac{447.3}{1 + (N/6.3 \times 10^{16})^{0.76}} \text{cm}^2/\text{V-s}
\]

(2.36)

Less specialized impurities as well as crystal defects will decrease mobilities further, for similar reasons.

Increasing temperature will increase the vibration of the host atoms, making them larger “targets,” again decreasing the average time between collisions as well as the carrier mobilities. This effect becomes less pronounced at high doping levels, where ionized dopants are effective carrier scatterers.

Increasing the strength of the electric field will eventually increase the drift velocities of carriers to values where they will become comparable to the random thermal velocities. Hence, the total velocity of electrons will ultimately increase with field strength, decreasing the time between collisions and the mobility.
2.14.2 Diffusion

Apart from motion by drift, carriers in semiconductors can also flow by diffusion. It is a well-known physical effect that any excess concentration of particles such as gas molecules will tend to dissipate itself unless constrained. The basic cause of this effect is the random thermal velocity of the particles involved.

The flux of particles is proportional to the negative of the concentration gradient (Fig. 2.17). Since current is proportional to the flux of charged particles, the current density corresponding to a one-dimensional concentration gradient of electrons is

\[ J_e = qD_e \frac{dn}{dx} \quad (2.37) \]

where \( D_e \) is a constant known as the diffusion constant. Similarly, for holes

\[ J_h = -qD_h \frac{dp}{dx} \quad (2.38) \]

Note the sign difference between Eqs. (2.37) and (2.38), which is due to the opposite types of charges involved. Drift and diffusion processes are fundamentally related and the mobilities and diffusion constants are not independent. They are interconnected by the Einstein relations

\[ D_e = \frac{kT}{q \mu_e} \quad \text{and} \quad D_h = \frac{kT}{q \mu_h} \quad (2.39) \]

![Figure 2.17. Diffusive flux of carriers in the presence of a concentrator gradient.](image)

\[ kT/q \] is a parameter that will appear often in relation to solar cells. It has the dimensions of voltage and the value of 26 mV at room temperature, a value worth committing to memory!

2.15 SUMMARY

The major points arising from this chapter are the following. Semiconductors have an electronic structure such that one band of allowed states virtually completely occupied by electrons (the valence band) is separated by a forbidden energy gap from the next band of allowed states, which is virtually devoid of electrons (the conduction band). Current flow in semiconductors is due to both motion of electrons in the conduction band and the effective motion of vacancies or holes in the valence band. In many situations, electrons in the conduction band and holes in the valence band can be regarded as free particles, provided that an "effective" mass is used to include the effect of periodic forces of the host atoms in the crystal. Most conduction-band electrons have energies close to that of the conduction-band edge, whereas most holes have energies close to that of the valence-band edge.

Semiconductors can be divided into "direct"- and "indirect"-band-gap types, depending on the form of the relationship between the energy of electrons in the conduction band and their crystal momentum.

Specialized impurities known as dopants, when introduced into semiconductors, can control the relative concentrations of electrons in the conduction band of a semiconductor and holes in the valence band. Carriers in these bands can flow by drift and diffusion when the appropriate perturbations are present.

In Chapter 3, additional electronic processes occurring within semiconductors when disturbed by light are described. From the fundamental mechanisms discussed in this and the next chapter, a single system of self-consistent equations will be synthesized. This system will be used in later chapters to establish the principles of solar cell design.

**EXERCISES**

2.1. For a crystal with a cubic unit cell, indicate on a sketch of the cell the following crystal planes: (a) (100); (b) (010); (c) (110); (d) (111).

2.2. (a) Silicon solar cell performance can be improved by selectively etch-
sition to an empty lower-energy state with all (or most) of the energy difference between states emitted as light. All the mechanisms considered for absorption have inverse radiative recombination processes (Fig. 3.9). Radiative recombination occurs more rapidly in direct-band-gap semiconductors than in indirect types because a two-step process involving a phonon is required for the latter.

The total radiative recombination rate, $R_R$, is proportional to the product of the concentration of occupied states (electrons) in the conduction band and that of unoccupied states in the valence band (holes):

$$R_R = Bnp$$  \hspace{1cm} (3.17)

where $B$ is a constant for a given semiconductor. Because of the relationship between optical absorption and this recombination process, $B$ can be calculated from the semiconductor's absorption coefficient (Ref. 3.2).

In thermal equilibrium when $np = n_0^2$, this recombination rate is balanced by an equal and opposite generation rate. In the absence of generation by external stimuli, the net recombination rate corresponding to Eq. (3.17), $U_R$, is given by the total recombination rate minus the equilibrium generation rate:

$$U_R = B(np - n_0^2)$$  \hspace{1cm} (3.18)

3.4 RECOMBINATION PROCESSES

3.4.1 Relaxation to Equilibrium

Light of appropriate wavelength shining on a semiconductor creates electron-hole pairs. The concentrations of carriers in illuminated material will therefore be in excess of their values in the dark. If the light is switched off, these concentrations decay back to their equilibrium values. The process by which this decay occurs is known as recombination. Three different recombination mechanisms will be described in the following sections. These mechanisms can occur in parallel, in which case the recombination rate is just the sum of those for the individual processes.

3.4.2 Radiative Recombination

Radiative recombination is just the reverse of the absorption process described in Section 3.3. An electron occupying a higher-energy state than it would under thermal equilibrium makes a tran-
With any recombination mechanism, it is possible to define associated carrier lifetimes, $\tau_e$ and $\tau_h$, for electrons and holes:

$$\tau_e = \frac{\Delta n}{U}$$

$$\tau_h = \frac{\Delta p}{U}$$

(3.19)

where $U$ is the net recombination rate and $\Delta n$ and $\Delta p$ are the disturbances of the respective carriers from their equilibrium values, $n_0$ and $p_0$.

For the radiative recombination mechanism with $\Delta n = \Delta p$, the characteristic lifetime determined from Eq. (3.18) is (Ref. 3.2)

$$\tau = \frac{n_0 p_0}{Bn_e^2 (n_0 + p_0)}$$

(3.20)

For silicon, $B$ has the value of about $2 \times 10^{-15}$ cm$^3$/s (Ref. 3.2).

As might be expected, radiative recombination lifetimes are much smaller in direct-band-gap material than in indirect material. This process forms the basis of commercial semiconductor lasers and light-emitting diodes using GaAs and its alloys. For silicon, other recombination mechanisms are far more important.

### 3.4.3 Auger Recombination

In the Auger (pronounced something like "oh-shay") effect, the electron recombining with the hole gives the excess energy to a second electron (either in the conduction or valence band) instead of emitting light. This process is illustrated in Fig. 3.10. This second electron then relaxes back to its original energy by emitting phonons. Auger recombination is just the reverse process of the more familiar impact ionization effect, where a high-energy electron collides with an atom, breaking a bond and creating an electron–hole pair. The characteristic lifetime $\tau$ associated with the Auger process is (Ref. 3.2)

$$\frac{1}{\tau} = C n p + D n^2$$

or

$$\frac{1}{\tau} = C n p + D p^2$$

(3.21)

Figure 3.10. Auger recombination with associated excess energy given to an electron in:
(a) Conduction band
(b) Valence band.

for materials with an abundance of electrons and holes, respectively. The first term on the right in each case describes electron excitation in the minority carrier band, and the second describes it in the majority carrier band. Auger recombination is particularly effective in relatively highly doped material due to this second term. For good-quality silicon, this is the dominant recombination process for impurity levels greater than $10^{17}$ cm$^{-3}$. The experimental results for the variation of lifetime with increasing dopant density for high-quality silicon shown in Fig. 3.11 indicate the rapid decrease at high dopant density due to Auger processes.

### 3.4.4 Recombination through Traps

It has been indicated in Chapter 2 that impurities and defects in semiconductors can give rise to allowed energy levels within the otherwise forbidden gap. These defect levels create a very efficient two-step recombination process whereby electrons relax from conduction-band energies to the defect level and then relax to the valence band, annihilating a hole as shown in Fig. 3.12(a).
The analysis of the dynamics of this process is straightforward but lengthy (Ref. 3.4). The result is that the net recombination-generation rate by traps, $U_T$, can be written

$$U_T = \frac{np - n_1^2}{\tau_{h0}(n + n_1) + \tau_{e0}(p + p_1)}$$  \hspace{1cm} (3.22)

where $\tau_{h0}$ and $\tau_{e0}$ are lifetime parameters whose values depend on the type of trap and the volume density of trapping defects. $n_1$ and $p_1$ are parameters arising from the analysis which introduce a dependency of the recombination rate upon the energy of the trapping level, $E_t$:

$$n_1 = N_C \exp \left( \frac{E_t - E_C}{kT} \right)$$  \hspace{1cm} (3.23)

$$n_1 p_1 = n_1^2$$  \hspace{1cm} (3.24)

Equation (3.23) is very similar in form to the expression for the electron concentration in terms of the Fermi-level energy as given by Eqs. (2.14) and (2.15). If $\tau_{e0}$ and $\tau_{h0}$ are of the same order of magnitude, it is not difficult to show that $U$ will have its peak value when $n_1 \approx p_1$. This occurs if the defect level lies near the middle of the forbidden band gap. Therefore, impurities that introduce energy levels near midgap are very effective recombination centers.

**3.4.5 Recombination at Surfaces**

Surfaces represent rather severe defects in the crystal structure and are the site of many allowed states within the forbidden gap, as indicated in Fig. 3.12(b). Recombination can therefore occur very efficiently at surfaces by the mechanism described in 3.4.4. The net recombination rate per unit area, $U_A$, for a single-level surface state takes a form similar to Eq. (3.22):

$$U_A = \frac{S_{e0} S_{h0} (np - n_1^2)}{S_{e0} (n + n_1) + S_{h0} (p + p_1)}$$  \hspace{1cm} (3.25)

where $S_{e0}$ and $S_{h0}$ are surface-recombination velocities. Again, surface-state levels lying near midgap are the most effective recombination centers.
3.5 BASIC EQUATIONS
OF SEMICONDUCTOR-DEVICE PHYSICS

3.5.1 Introduction

In previous sections, the relevant properties of semiconductors have been reviewed. This material will now be consolidated by synthesizing from it a set of basic equations capable of describing the operation of semiconductor devices. Solution of these equations allows the ideal characteristics of most semiconductor devices, including solar cells, to be determined. The equations will be written in one-dimensional form, with variations in the other two spatial dimensions neglected. Their three-dimensional form is similar except that spatial derivatives are replaced by the divergence operator for vector quantities (electric field, current density) and by the gradient operator for scalar quantities (concentrations, potentials).

3.5.2 Poisson’s Equation

The first equation in the system may be familiar from electrostatics. It is Poisson’s equation, one of Maxwell’s equations (Ref. 3.5) which relates the divergence of the electric field to the space charge density, \( \rho \). In one dimension, it takes the form

\[
\frac{d\xi}{dx} = \frac{\rho}{\varepsilon} \tag{3.26}
\]

where \( \varepsilon \) is the material’s permittivity. This equation is a differentiated form of Gauss’s law, which may be more familiar.

Looking at the contributors to charge density in a semiconductor, electrons in the conduction band contribute a negative charge whereas holes give a positive charge. A donor impurity that is ionized (i.e., has its extra electron removed) has a net positive charge due to the unneutralized extra positive charge at the nucleus. Similarly, an ionized acceptor contributes a negative charge. Hence,

\[
\rho = q(p - n + N_D^+ - N_A^-) \tag{3.27}
\]

where \( p \) and \( n \) are the densities of holes and electrons, and \( N_D^+ \) and \( N_A^- \) are the densities of ionized donors and acceptors. Less specialized impurities and defects can also act as charge-storage centers, so corresponding terms should be included in Eq. (3.27). However, the volume density of such nonidealties is kept as small as possible in solar cell work, making the charge contributions relatively insignificant.

As mentioned in Chapter 2, most donors and acceptors are ionized under normal conditions, so that

\[
N_D^+ \approx N_D \quad N_A^- \approx N_A \tag{3.28}
\]

where \( N_D \) and \( N_A \) are the total density of donors and acceptors.

3.5.3 Current-Density Equations

In Chapter 2, it was seen that electrons and holes could contribute to current flow by drift and diffusion processes. Hence, the expressions for the total current densities of electrons and holes, \( J_e \) and \( J_h \), become

\[
J_e = q\mu_e n \xi + qD_e \frac{dn}{dx} \tag{3.29}
\]

\[
J_h = q\mu_h p \xi - qD_h \frac{dp}{dx}
\]

The mobilities and diffusion constants are related through the Einstein relationships \( \left[ D_e = (kT/q)\mu_e; D_h = (kT/q)\mu_h \right] \).

3.5.4 Continuity Equations

The final equations in the system are “bookkeeping”-type equations which merely keep track of the number of electrons and holes in a system and ensure that none materialize “out of thin air.”

Referring to the elemental volume of length \( \delta x \) and cross-sectional area \( A \) of Fig. 3.13, it can be stated that the net rate of increase of electrons in this volume is the rate at which they enter minus the rate at which they exit plus the rate at which they are generated in this volume minus the rate at which they recombine.

![Figure 3.13. Elemental volume for deriving the continuity equations for electrons.](image-url)
But the rates of entering and exiting are proportional to the current densities at the respective faces of the elemental volume. Hence,

\[
\text{rate of entering} - \text{rate exiting} = \frac{A}{q} \left( -J_e(x) - [-J_e(x + \delta x)] \right)
\]

\[
= A \frac{dJ_e}{dx} \delta x
\]

(3.30)

rate of generation - rate of recombination

\[
= A \delta x (G - U)
\]

(3.31)

where \( G \) is the net generation rate by external processes such as illumination by light, and \( U \) is the net recombination rate. Under steady-state conditions the net rate of increase must be zero, so that

\[
\frac{1}{q} \frac{dJ_e}{dx} = U - G
\]

(3.32)

Similarly, for holes

\[
\frac{1}{q} \frac{dJ_h}{dx} = -(U - G)
\]

(3.33)

3.5.5 Equation Set

The set of basic equations is

\[
\frac{d\xi}{dx} = \frac{q}{e} (p - n + N_D - N_A)
\]

\[
J_e = q \mu_e n \xi + q D_e \frac{dn}{dx}
\]

\[
J_h = q \mu_h p \xi - q D_h \frac{dp}{dx}
\]

(3.34)

\[
\frac{1}{q} \frac{dJ_e}{dx} = U - G
\]

\[
\frac{1}{q} \frac{dJ_h}{dx} = -(U - G)
\]

Subsidiary relations are also required for \( U \) and \( G \). Expressions for these terms depend on the specific processes involved.

The Equations (3.34) form a coupled set of nonlinear differential equations for which it is not possible to find general analytical solutions. They can be solved numerically on a digital computer to give ideal properties for a range of possible semiconductor-device configurations. Examples of this technique applied to solar cells are contained in Refs. 3.6 to 3.8. It is also possible to get good solutions to these equations much more simply and with more insight into the physical principles involved by making a series of well-thought-out approximations. This technique is demonstrated in Chapter 4.

3.6 SUMMARY

Light made up of photons of energy larger than the forbidden band gap can be absorbed in a semiconductor by creating electron-hole pairs. In direct-band-gap semiconductor, the light is absorbed quickly. In indirect-band-gap materials, the emission or absorption of a phonon also is required for photon energies near that of the band gap. Hence, indirect-band-gap material absorbs weakly for such energies but becomes strongly absorbing for higher energies where direct transitions are also possible.

Recombination of carrier concentrations in excess of equilibrium values can occur through a variety of processes. Radiative recombination is the reverse of light absorption and is an important mechanism for direct-band-gap semiconductors. Auger recombination is important at high doping concentrations, whereas recombination through traps caused by impurities and defects is important for indirect-band-gap semiconductors and for those with an undeveloped supporting technology. These recombination processes occur in parallel. The total recombination rate is just the sum of the individual rates. The inverse of the net recombination lifetime is the sum of the inverses of the individual lifetimes. Recombination also occurs particularly effectively at semiconductor surfaces.

The end result of our review of the properties of semiconductors and the starting point for the analysis of the properties of solar cells is a system of coupled differential equations relating the spatial distribution of the quantities important in determining the internal operation of solar cells. Techniques for obtaining solutions to these equations are discussed in Chapter 4.

EXERCISES

3.1. Monochromatic light is normally incident on a flat silicon surface. Using the data of Fig. 3.1, calculate the fraction reflected at the following