ME 432 Fundamentals of Modern Photovoltaics
Homework Assignment #2, Fall 2020
Due Monday September 21

In this assignment, we will explore solar array outputs a little further, and practice working with the solar spectrum incident on earth.

Please note: You may attach Excel spreadsheets, Matlab code, Mathematica, etc. However, if you use external software, please write out the formulas or methodology you used to calculate your answer. For all problems, your logic flow/thought process are to be described clearly to obtain full credit.

Question #1: PV on Roofs vs. PV on Cars

(a) Calculate the space required in Illinois to meet the time-averaged power requirements of a typical home. Assume a 15% efficient PV system, and that the energy burn rate of an average home is around 2kW.

Solution:

note that 2 kW = 2 kWh/h = 48 kWh/day

land required = (power needed)/(solar resource * efficiency)

= (48 kWh/day)/(4.2 kWh/m^2/day * 0.15)

= 76.19 m^2

(b) How do these values change, if peak demand must be met (i.e. there is no energy storage battery available)? You can assume that peak energy is roughly double the average load, so 4kW.

Solution:

Twice the power needed means twice the area requirements.

Land required = 152.38 m^2

Better Solution:

Assume that peak power is needed in the middle of the day, i.e. when the sun is highest in the sky and the irradiance is around 1000 W/m^2.

Land required = (power needed)/(solar resource * efficiency)

= (4000 W)/(1000 W/m^2 * 0.15)

= 26.7 m^2

* note how this area is actually SMALLER than the area in part (a)

(c) Repeat parts (a) and (b) for an electric car. To estimate the electricity consumption for a car, we'll assume the fuel efficiency and peak power of a Tesla roadster (vroom!!!).

Solution:

To meet the average burn rate:

land required = (power needed)/(solar resource * efficiency)

= (12 kWh/day)/(4.2 kWh/m^2/day * 0.15)

= 19.05 m^2

To power the engine at full throttle:

land required = (power needed)/(solar resource * efficiency)

= (215*24 kWh/day)/(4.2 kWh/m^2/day * 0.15)

= 8190.48 m^2

Alternative Solution: To power the engine at full throttle, assuming you do it during peak sun: land required = (power needed)/(solar resource * efficiency)

= (215000 W)/(1000 W/m^2 * 0.15)

= 1430 m^2

(d) Given your answers to (a), (b), and (c), where do you think PV will be deployed – on cars or on rooftops?

Solution:

Clearly there is not enough roof space for solar panels even to power the car using storage and meeting the car's average power. Homes, which have a much larger roof area, are clearly more suitable for PV deployment.

(e) How much larger would your home array have to be if, in addition to meeting your home's time - averaged power requirements, you also plan to charge your Tesla battery using your home rooftop PV system while its sitting in the garage?

Solution:

You must add the amount of power needed to drive the Tesla per day, which will be the same amount that will be needed to charge it in your home.

land required = (power needed)/(solar resource * efficiency)

= (7.1 kWh/day + 48 kWh/day)/(4.2 kWh/m^2/day * 0.15)

= 87.46 m^2

You will want to make use of the following information:

- The 2020 Tesla Roadster standard issue comes with a 200 kWh battery pack. The engine rating is not yet released by Tesla, but let's assume 215 kW which is the rating of the current Roadster 2.5 Sport. The efficiency is reported to be around 32.3 kWh per 100 miles.

- The average car drives around 13,500 miles a year (source: epa.gov), which means the average person drives around 37 miles a day and would consume 12.0 kWh per day in their car if they drove the Tesla roadster.

Question #2: Integrated Spectral Irradiance for AM1.5.
Calculate numerically the integrated spectral irradiance in W/m^2 for the AM1.5 solar spectra. (Note that we already know what the answers should be (see your class notes!), but we will make use of this calculation in problem 3 and 4)

The spectra can be downloaded from:
http://rredc.nrel.gov/solar/spectra/am1.5/
Use the "Direct+Circumsolar" column.

Hint: Watch your units here, and make sure to note that the data you download (Intensity vs. Wavelenth) is not necessarily provided on uniform intervals. You may need to go back to your calculus books to recall the trapezoidal rule or Simpson's rule.

Solution:
The total power density can be determined by integrating the spectral irradiance over all wavelengths:

$$(\text{power/area}) = \int_0^{\lambda_{\min}} I(\lambda)\,d\lambda$$

where I(λ) is the spectral intensity, which is given in W*m^{-2}*nm^{-1}. Using numerical integration gives a total power density around 901 W/m^2.

Question #3: Solar photon spectrum.
Using the same "Direct+Circumsolar" data, make your own plot of the photon spectral flux vs. wavelength. You will need to convert from Intensity (W m^{-2} nm^{-1}) instead to (#photons m^{-2} s^{-1} nm^{-1}).

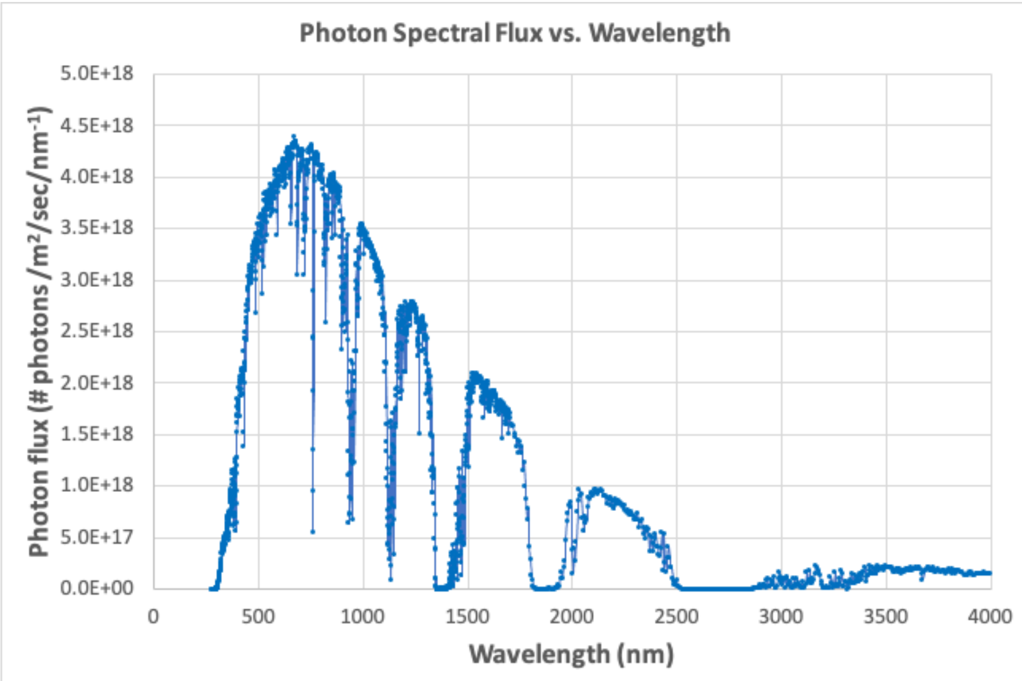
Solution:
All that needs to be done here is to convert the intensity data to the number of photons. First, we use the following expression to relate the energy of a photon to the wavelength:

$$E(\lambda) = \frac{hc}{\lambda}$$

You can set h = 6.62607(10^{-34}) J-s for Planck's constant, and c = 3(10^8) m/s = 3(10^{17}) nm/s. That is the energy of a single photon for a given wavelength. Then the photon flux at a given wavelength is given by

(# photons m^{-2} s^{-1} nm^{-1}) = (total energy m^{-2} s^{-1} nm^{-1})/(energy per photon)

The resulting plot is:



Question #4: Reflection and Absorption Losses.
For many of these problems, you may need to look up the index of refraction or absorption coefficient for silicon and silicon nitride (SiN_x). Your class discussion slides, or on http://www.pveducation.org/PVCDROM is a good place to look. You can assume that the absorption for a thin SiN_x layer is negligible.

(a) For 550 nm light (peak of the solar spectrum), what percentage of light is reflected off the front surface of a polished silicon wafer?

Solution:

The index of refraction of silicon around 550 nm is 4.1; for air the index of refraction is 1.
$R = (n_{Si} - n_{air})^2/(n_{Si} + n_{air})^2 = 0.37$

(b) If SiN_x is used as an anti-reflection coating (ARC), what thickness should be used to optimize for 550 nm light?

Solution:

You need to use the index of refraction for silicon nitride, provided in the class notes (around n=2.0). The thickness is given by

$$thickness = \frac{\lambda}{4n} = \frac{550nm}{4(2)} = 68.75nm$$

(c) If we assume only one pass of light through the silicon, estimate the thickness required to absorb 90% of incident, non-reflected photons at 1070 nm.

Solution:

Using the Beer-Lambert law, we have

$$I = I_o \exp(-\alpha x)$$

$$\frac{I}{I_o} = 0.1 = \exp(-8x)$$

$$x = 0.288 \text{ cm}$$

where I have estimated the absorption coefficient of silicon at 1070 nm to be around 8 cm^{-1} (from plots in class discussion slides).

(d) Say we texture the back surface, and introduce total internal reflection at the front surface. It turns out that there is a fundamental upper limit to how much the effective path length of light can be extended. This limit arises from pure statistical mechanics considerations, and is known as the Yablonovitch limit. It states that the effective path length of the light can only be increased by a factor of $4n^2$, where n is the refractive index. How does the thickness needed in part (c) change, assuming we achieve this limit?

Solution:

The refractive index for silicon around 1070 nm is around n=3.55. Thus, the effective path length increases by a factor of $4n^2 = 50.41$. The absorber layer can be 50.41 times thinner, giving x = (0.288/50.41) cm = 57.13 μm.

Note: If you are curious about the Yablonovitch limit, for more details see *E. Yablonovitch and G.D. Cody, IEEE Trans. Electron Dev. 29, 3000 (1982)*. The following websites may be useful:

http://optoelectronics.eecs.berkeley.edu/ey1982josa727.pdf
http://www.eecs.berkeley.edu/Faculty/Homepages/yablonovitch.html